

İNTEGRAL-2

BASİT DEĞİŞKEN DEĞİŞTİRME

BELİRSİZ İNTEGRAL ALMA YÖNTEMLERİ 1

DEĞİŞKEN DEĞİŞTİRME METODU

$\int f(x)dx$ ifadesini hesaplayabilmek için sıklıkla türevde zincir kuralını tersine doğru işletiriz.

Yani $\int f(g(x)).g'(x)dx$ integrali $\int f(u)du$ biçiminde yazılır ve integral alınarak $F(u)+C=F(g(x))+C$ olur.

Değişken değiştirme dediğimiz bu yöntemi kısaca özetlersek.

Adım1 $\int f(g(x)).g'(x)dx$ ifadesinde $u=g(x)$ ve $du=g'(x)dx$ seçilerek $\int f(u).du$ elde edilir. (türevi var olan fonksiyona u der ve du yu hesaplayıp integrali u değişkenine göre tekrar yazılır)

Adım2 u değişkenine göre integral alınır

Adım3 sonuçta u yerine yazılır.

Örnek...1 :

$$\int (x^5+x^2)(5x^4+2x)dx=?$$

$$u = x^5+x^2 \quad du = (5x^4+2x)dx$$

$$\int u du = \frac{u^2}{2} + C = \frac{(x^5+x^2)^2}{2} + C$$

Örnek...2 :

$$\int \frac{2x+7}{x^2+7x+121} dx \quad u = x^2+7x+121$$

$$du = (2x+7)dx$$

$$\int \frac{du}{u} = \ln|u| + C = \ln|x^2+7x+121| + C$$

Örnek...3 :

$$\int \cos(5x+1) dx \quad u = 5x+1 \quad du = 5dx$$

$$\int \cos u \cdot \frac{du}{5} = \frac{1}{5} \sin u + C$$

$$= \frac{1}{5} \sin(5x+1) + C$$

Örnek...4 :

$$\int (x^5+4x+e)^5 \cdot (5x^4+4) dx$$

$$u = x^5+4x+e \rightarrow du = (5x^4+4)dx$$

$$\int u^5 du = \frac{u^6}{6} + C = \frac{(x^5+4x+e)^6}{6} + C$$

Örnek...5 :

$$\int 10 \sin(5x^3+1) x^2 dx$$

$$u = 5x^3+1 \rightarrow du = 15x^2 dx$$

$$\int 10 \sin u \cdot \frac{du}{15} = \frac{1}{3} (-\cos u) + C$$

$$= -\frac{\cos(5x^3+1)}{3} + C$$

Örnek...6 :

$$\int (3x-19)^{23} dx \quad u = 3x-19 \quad du = 3dx$$

$$\int u^{23} \frac{du}{3} = \frac{1}{3} \frac{u^{24}}{24} + C = \frac{(3x-19)^{24}}{72} + C$$

Örnek...7 :

$$\int \frac{x}{\cos^2(x^2-2)} dx \quad u = x^2-2 \quad du = 2x dx$$

$$\int \frac{du/2}{\cos^2 u} = \frac{1}{2} \tan u + C = \frac{1}{2} \tan(x^2-2) + C$$

Örnek...8 :

$$\int e^{x^2+\pi} 2x dx \quad u = x^2+\pi \quad du = 2x dx$$

$$\int e^u du = e^u + C = e^{x^2+\pi} + C$$

Örnek...9 :

$$\int \frac{1}{(x+2)^{15}} dx \quad u = x+2 \Rightarrow du = dx$$

$$\int \frac{du}{u^{15}} = \frac{u^{-14}}{-14} = -\frac{1}{14 \cdot (x+2)^{14}} + C$$

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Örnek...10 :

$$\int \frac{1}{x \cdot \ln x} dx \quad u = \ln x \quad du = \frac{1}{x} dx$$

$$\int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C$$

Örnek...11 :

$$\int \frac{1}{\ln x} dx = \int \frac{dx}{x \cdot \ln x} = \ln|\ln x| + C$$

Örnek...12 :

$$\int \frac{dx}{x \cdot (1 + \ln^2 x)} \quad u = \ln x \quad du = \frac{1}{x} dx$$

$$\int \frac{du}{1+u^2} = \arctan u + C$$

$$= \arctan(\ln x) + C$$

Örnek...13 :

$$\int \frac{1}{x \cdot (1 + \ln^2 x)} dx \quad u = \ln x \quad du = \frac{1}{x} dx$$

$$\int \frac{du}{1+u^2} = \ln|1+u| + C$$

$$= \ln|1 + \ln x| + C$$

Örnek...14 :

$$\int \sqrt{x+2} dx \quad u = x+2 \quad du = dx$$

$$\int \sqrt{u} du = \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{3} \cdot \sqrt{(x+2)^3} + C$$

Örnek...15 :

$$\int \sqrt{6x-23} dx \quad u = 6x-23 \quad du = 6 dx$$

$$\int \sqrt{u} \cdot \frac{du}{6} = \frac{1}{6} \cdot \frac{u^{3/2}}{3/2} = \frac{1}{9} \cdot \sqrt{6x-23}^3 + C$$

Örnek...16 :

$$\int \frac{5x}{\sqrt{6x^2-23}} dx \quad u = 6x^2-23 \Rightarrow du = 12x dx$$

$$\int \frac{5 \cdot du/12}{\sqrt{u}} = \frac{5}{12} \cdot \frac{u^{1/2}}{1/2} = \frac{5}{6} \sqrt{6x^2-23} + C$$

Örnek...17 :

$$\int \frac{2x}{\sqrt{8x+3}} dx \quad u = 8x+3 \Rightarrow du = 8 dx$$

$$\int \frac{\left(\frac{u-3}{8}\right) \cdot \frac{du}{8}}{\sqrt{u}} = \frac{1}{32} \int \frac{u-3}{\sqrt{u}} du$$

$$= \frac{1}{32} \left(\frac{u^{3/2}}{3/2} - 3 \cdot \frac{u^{1/2}}{1/2} \right)$$

$$= \frac{1}{48} \cdot \sqrt{8x+3}^3 - 6 \cdot \sqrt{8x+3} + C$$

Örnek...18 :

$$\int \sin^2 x \cdot \cos x dx \quad u = \sin x \quad du = \cos x dx$$

$$\int u^2 du = \frac{u^3}{3} + C \rightarrow \frac{\sin^3 x}{3} + C$$

Örnek...19 :

$$\int e^{\sin 2x} \cdot \cos 2x dx \quad u = \sin 2x \quad du = \cos 2x \cdot 2 dx$$

$$\int \frac{e^u du}{2} = \frac{1}{2} e^u + C \rightarrow \frac{1}{2} e^{\sin 2x} + C$$

$$\frac{1}{2} \cdot \frac{1}{2} e^{\sin 2x} = \frac{1}{4} e^{\sin 2x} \Rightarrow du = e^{\sin 2x} \cdot \cos 2x \cdot 2 dx$$

$$\int \frac{du}{2} = \frac{1}{2} u + C = \frac{1}{2} e^{\sin 2x} + C$$

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Örnek...20 :

$$\int \frac{1}{a^2+x^2} dx \quad \left(\frac{dx}{a^2(1+(\frac{x}{a})^2)} \right) = \frac{1}{a^2} \int \frac{dx}{1+(\frac{x}{a})^2}$$

$$u = \frac{x}{a} \quad du = \frac{1}{a} dx \rightarrow \frac{1}{a^2} \int \frac{a du}{1+u^2} = \frac{1}{a} \int \frac{du}{1+u^2}$$

$$\frac{1}{a} \arctan u = \frac{1}{a} \arctan \left(\frac{x}{a} \right) + C$$

Örnek...21 :

$$\int \frac{1}{144+x^2} dx = \frac{1}{12} \arctan \frac{x}{12} + C$$

Örnek...22 :

$$\int \frac{3x+13}{x^2+9} dx \quad \left(\frac{3x}{x^2+9} + \frac{13}{x^2+9} \right) dx \quad \begin{matrix} u=x^2+9 & du=2x dx \\ \int \frac{3/2 du}{u} \end{matrix}$$

$$\frac{3}{2} \ln |x^2+9| + \frac{13}{3} \arctan \frac{x}{3} + C$$

Örnek...23 :

$$\int \frac{\arctan x}{1+x^2} dx \quad u = \arctan x \quad du = \frac{1}{1+x^2} dx$$

$$\int u du = \frac{u^2}{2} + C = \frac{\arctan^2 x}{2} + C$$

Örnek...24 :

$$\int \frac{1}{x^2+16x+65} dx \quad \left(\frac{dx}{(x+8)^2+1} \right) \quad \begin{matrix} u=x+8 \\ du=dx \end{matrix}$$

$$\int \frac{du}{1+u^2} = \arctan u + C = \arctan(x+8) + C$$

Örnek...25 :

$$\int \frac{1}{16x^2-8x+2} dx \quad u=4x-1 \quad du=4 dx$$

$$\frac{1}{(4x-1)^2+1}$$

$$\int \frac{du/4}{1+u^2} = \frac{1}{4} \arctan(u) + C$$

Örnek...26 :

$$\int \frac{x^5}{1+x^{12}} dx \quad u = x^6 \rightarrow du = 6x^5 dx$$

$$\int \frac{du/6}{1+u^2} = \frac{1}{6} \arctan u + C$$

$$= \frac{1}{6} \arctan(x^6) + C$$

Örnek...27 :

$$\int \frac{1}{\sqrt{a^2-x^2}} dx \quad \left(\frac{dx}{\sqrt{a^2(1-(\frac{x}{a})^2)} \right) = \frac{1}{a} \int \frac{dx}{\sqrt{1-(\frac{x}{a})^2}}$$

$$u = \frac{x}{a} \quad du = \frac{1}{a} dx \quad \frac{1}{a} \int \frac{a du}{\sqrt{1-u^2}} = \arcsin u + C$$

$$= \arcsin \left(\frac{x}{a} \right) + C$$

Örnek...28 :

$$\int \frac{1}{\sqrt{256-x^2}} dx \quad \arcsin \frac{x}{16} + C$$

Örnek...29 :

$$\int \frac{1}{\sqrt{12-(x+1)^2}} dx \quad u=x+1 \quad du=dx$$

$$\int \frac{du}{\sqrt{12-u^2}} = \arcsin \frac{u}{2\sqrt{3}} + C$$

$$= \arcsin \left(\frac{x+1}{2\sqrt{3}} \right) + C$$

Örnek...30 :

$$\int \frac{1}{\sqrt{-x^2-2x+3}} dx \quad u=x+1 \quad du=dx$$

$$\int \frac{du}{\sqrt{4-u^2}} = \arcsin \frac{u}{2} + C$$

$$= \arcsin \left(\frac{x+1}{2} \right) + C$$

Örnek...31 :

$$\int f(x) dx = \tan x \quad \text{ise } \int f\left(\frac{x}{2}\right) dx = ? \quad u = \frac{x}{2} \quad du = \frac{dx}{2}$$

$$\int f(u) 2 du = 2 \int f(u) du = 2 \tan u = 2 \tan x + C$$

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Örnek...32 :

$\int f'(x) \cdot f''(x) dx$ integralinin $f(x)$ türünden eşiti nedir?

$$u = f'(x) \quad du = f''(x) dx$$

$$\int u \cdot du = \frac{u^2}{2} + C = \left[\frac{f'(x)}{2} \right]^2 + C$$

Örnek...33 :

Aşağıda verilen değişken değiştirmeleri yaparak integralleri tekrar yazınız

1) $\int \frac{e^x}{\sqrt{16-e^{2x}}} dx$, $a=e^x$

$$u=e^x \quad du=e^x dx$$

$$\int \frac{du}{\sqrt{16-u^2}}$$

2) $\int \frac{1}{\sqrt{4-x^2}} dx$, $x=2\sin t$

$$x=2\sin t \rightarrow 4-x^2=4-4\sin^2 t \\ dx=2\cos t dt = 4(1-\sin^2 t) \\ = 4\cos^2 t$$

$$\int \frac{2\cos t dt}{\sqrt{4\cos^2 t}}$$

$$\int \frac{2\cos t dt}{|2\cos t|} \rightarrow \int \frac{dt}{1} = \int dt$$

3) $\int \frac{1}{\sqrt{25x^2-1}} dx$, $x=5\sec t = \frac{5}{\cos t}$

$$dx = \frac{5 \sin t}{\cos^2 t} dt$$

4) $\int \frac{dx}{\sqrt{4+x^2}}$, $x=2\tan t$ $\rightarrow 4+x^2=4+4\tan^2 t$
 $dx=2(1+\tan^2 t) dt$

$$\int \frac{2(1+\tan^2 t) dt}{\sqrt{4+4\tan^2 t}} = \frac{2(1+\tan^2 t) dt}{2\sqrt{1+\tan^2 t}}$$

$$1+\tan^2 t = \frac{1}{\cos^2 t} \Rightarrow \int \frac{2}{\cos^2 t} dt = \int \sec t dt$$

5) $\int \frac{\sqrt{\ln x}}{x} dx$, $a=\ln x$
 $da = \frac{1}{x} dx$

$$\int \sqrt{a} da$$

6) $\int \left(\frac{\sqrt{3x-1}}{\sqrt[3]{3x-1} + \sqrt{3x-1}} \right) dx$, $\sqrt[3]{3x-1}=t$

$$3x-1=t^3$$

$$3dx=3t^2 dt$$

$$\int \left(\frac{t^6}{t^4+3} \right) \cdot 3t^2 dt$$

$$= 4 \int \frac{t^4}{t^4+3} dt$$

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BASİT DEĞİŞKEN DEĞİŞTİRME

DEĞERLENDİRME

1) $\int (4x^3 - 5x)(12x^2 - 5) dx = ?$

$$u = 4x^3 - 5x \quad du = (12x^2 - 5) dx$$

$$\int u du = \frac{u^2}{2} + C = \frac{(4x^3 - 5x)^2}{2} + C$$

2) $\int \cos(x^6 + 1)(x^5) dx = ?$

$$u = x^6 + 1 \quad du = 6x^5 dx$$

$$\int \cos u \cdot \frac{du}{6} = \frac{1}{6} \sin u + C = \frac{1}{6} \sin(x^6 + 1) + C$$

3) $\int (3x - 19)^{23} dx = ?$

$$u = 3x - 19 \quad du = 3 dx$$

$$\int u^{23} \frac{du}{3} = \frac{1}{3} \frac{u^{24}}{24} + C = \frac{1}{72} (3x - 19)^{24} + C$$

4) $\int \frac{1}{(6x - 5)^{11}} dx = ?$

$$u = 6x - 5 \quad du = 6 dx$$

$$\int \frac{du}{u^{11}} = \frac{1}{6} \frac{u^{-10}}{-10} = -\frac{1}{60} \frac{1}{(6x - 5)^{10}} + C$$

5) $\int \frac{e^{\ln x}}{x} dx = ?$

$$\int \frac{x}{x} dx = \int 1 dx = x + C$$

$u = \ln x$ ile derece bilirsiniz.

6) $\int \frac{1}{x(\sqrt{1 - \ln^2 x})} dx = ?$

$$u = \ln x \quad du = \frac{1}{x} dx$$

$$\int \frac{du}{u \sqrt{1 - u^2}} = \arcsin u + C = \arcsin(\ln x) + C$$

7) $\int \frac{3x+5}{(x^2+16)} dx = ?$

$$u = x^2 + 16 \quad du = 2x dx$$

$$\int \frac{3 \frac{du}{2} + 5}{u} = \frac{3}{2} \ln |u| + \frac{5}{4} \arctan \frac{x}{4} + C$$

8) $\int \frac{x}{(4x^4 + 100)} dx = ?$

$$u = 2x^2 \quad du = 4x dx$$

$$\int \frac{du/4}{u^2 + 100} = \frac{1}{4} \int \frac{du}{100 + u^2} = \frac{1}{4} \cdot \frac{1}{10} \arctan \frac{u}{10}$$

$$= \frac{1}{40} \arctan \left(\frac{2x^2}{10} \right) + C$$

9) $\int \frac{1}{(\sqrt{12x - 4x^2})} dx = ?$

$$u = 2x - 3 \quad du = 2 dx$$

$$\int \frac{du/2}{\sqrt{9 - u^2}} = \frac{1}{2} \arcsin \frac{u}{3} + C = \frac{1}{2} \arcsin \left(\frac{2x - 3}{3} \right) + C$$

10) $\int \frac{x^3}{(\sqrt{9 - x^2})} dx$ $x = 3 \sin t$ değişken değıştirmesi yapılırsa hangi integral elde edilir?

$$x = 3 \sin t \quad dx = 3 \cos t dt \quad \sqrt{9 - x^2} = \sqrt{9 - 9 \sin^2 t} = 3 \cos t$$

$$\int \frac{27 \sin^3 t \cdot 3 \cos t dt}{3 \cos t} = \int 27 \sin^3 t dt$$

11) $\int \left(\frac{\sqrt{5x+1} + x}{\sqrt{5x+1}} \right) dx$ integralinde $\sqrt{5x+1} = t$

değişken değıştirmesi yapılırsa hangi integral elde edilir?

$$5x + 1 = t^2 \quad 5 dx = t dt$$

$$\int \frac{t^2 + (t^2 - 1)/5}{t} \cdot \frac{t dt}{5} = \int \frac{20t^4 + t^2 - 1}{25} dt$$