

TÜREV ALMA KURALLARI

BÖLÜM 4

NOT

$y = \sin x \text{ ise}$

$y' = \cos x$

$y = \cos x \text{ ise}$

$y' = -\sin x$

$y = \tan x \text{ ise}$

$y' = (1 + \tan^2 x)$

$y' = \frac{1}{\cos^2 x} \text{ yada}$

$y' = \sec^2 x$

$y = \cot x \text{ ise}$

$y' = -(1 + \cot^2 x)$

$y' = -\frac{1}{\sin^2 x} \text{ yada}$

$y' = -\operatorname{cosec}^2 x$

TRİGONOMETRİK FONKSİYONLARIN TÜREVLERİ

1) $y = \sin u \rightarrow y' = u' \cdot \cos u$

2) $y = \cos u \rightarrow y' = -u' \cdot \sin u$

3) $y = \tan u \rightarrow$

$y' = u' \cdot (1 + \tan^2 u) = \frac{u'}{\cos^2 u} = u' \cdot \sec^2 u$

4) $y = \cot u \rightarrow$

$y' = -u' \cdot (1 + \cot^2 u) = \frac{-u'}{\sin^2 u} = -u' \cdot \operatorname{cosec}^2 u$

Örnek...1 :

$f(x) = \sin(x) \text{ ise } f'(x) = ?$

$f'(x) = \cos x //$

Örnek...2 :

$f(x) = \sin(x^2) \text{ ise } f'(x) = ?$

$f'(x) = (x^2)' \cos(x^2) = 2x \cdot \cos(x^2) //$

Örnek...3 :

$f(x) = \sin^2(x^2) \text{ ise } f'(x) = ?$

$f'(x) = 2 \cdot \sin(x^2) \cdot (\sin(x^2))'$

$f'(x) = 2 \cdot \sin(x^2) \cdot [2x \cos(x^2)]$

$f'(x) = 2 \cdot \sin(x^2) \cdot 2x \cdot \cos(x^2) \text{ (istenirse düzenlenebilir)}$

$f'(x) = \sin(2 \cdot x^2) \cdot 2x = 2x \cdot \sin(2x^2) //$

Örnek...4 :

$f(x) = \cos(5x+1) \text{ ise } f'(x) = ?$

$f'(x) = 5 \cdot (-\sin(5x+1))$

$f'(x) = -5 \cdot \sin(5x+1) //$

Örnek...5 :

$f(x) = x \cdot \cos(x^2-1) \text{ ise } f'(-1) = ?$

$f'(x) = 1 \cdot \cos(x^2-1) + x \cdot (2x) \cdot (-\sin(x^2-1))$

$f'(x) = \cos(x^2-1) - 2x^2 \cdot \sin(x^2-1)$

$f'(-1) = \cos(0) - 2 \cdot (-1)^2 \cdot \sin(0)$
 $= 1 - 2 \cdot 1 \cdot 0$
 $= 1 //$

Örnek...6 :

$f(x) = \cos^2(\sqrt{4x+1}) \text{ ise } f'(0) = ?$

$f'(x) = 2 \cdot \cos(\sqrt{4x+1}) \cdot (\cos(\sqrt{4x+1}))'$

$f'(x) = 2 \cdot \cos(\sqrt{4x+1}) \cdot (\sqrt{4x+1})' \cdot (-\sin \sqrt{4x+1})$

$f'(x) = 2 \cdot \cos(\sqrt{4x+1}) \cdot \frac{4}{2 \cdot \sqrt{4x+1}} \cdot (-\sin \sqrt{4x+1})$

$f'(0) = 2 \cdot \cos(1) \cdot \frac{4}{2 \cdot \sqrt{1}} \cdot (-\sin(1))$

Örnek...7 :

$f(x) = \tan(3x-1) \text{ ise } f'(x) = ?$

$f'(x) = 3 \cdot (1 + \tan^2(3x-1)) //$

 $\uparrow$
(3x-1)'in
turu

$f'(0) = -4 \cdot \sin(1) \cdot \cos(1)$

$= -2 \cdot 2 \cdot \sin(1) \cos(1)$

$= -2 \cdot \sin(2) //$

Örnek...8 :

$f(\cos^2 x) = 1 - \sin^2 x \text{ ise } f'(0) = ?$

$f(\cos^2 x) = \cos^2 x \text{ (x yerine } \cos^2 x \text{ yazılmış)}$

$f(x) = x \text{ (demek ki fonksiyon buymuş)}$

$f'(x) = 1$

$f'(0) = 1 //$

Örnek...9 :

$f(x) = 2x + \tan^2(4x^3) \text{ ise } f'(0) = ?$

$f'(x) = 2 + 2 \cdot \tan(4x^3) \cdot [\tan(4x^3)]'$

$f'(x) = 2 + 2 \cdot \tan(4x^3) \cdot (12x^2) \cdot (1 + \tan^2(4x^3))$

$f'(0) = 2 + 2 \cdot \tan(0) \cdot (12 \cdot 0) \cdot (1 + \tan^2(0))$

$f'(0) = 2 + 0$

$f'(0) = 2 //$

TÜREV - 4

(TRİGONOMETRİK VE TERS TRİGONOMETRİK FONKSİYON TÜREV ALMA KURALLARI)

Örnek...10 :

$$f(x) = \cot^2(4x) \text{ ise } f'\left(\frac{\pi}{16}\right) = ?$$

$$f'(x) = 2 \cdot \cot(4x) \cdot [\cot(4x)]'$$

$$f'(x) = 2 \cdot \cot(4x) \cdot [(-4) \cdot (1 + \cot^2(4x))]$$

$$f'\left(\frac{\pi}{16}\right) = 2 \cdot \cot\left(4 \cdot \frac{\pi}{16}\right) \cdot (-4) \cdot (1 + \cot^2\left(4 \cdot \frac{\pi}{16}\right))$$

$$f'\left(\frac{\pi}{16}\right) = 2 \cdot \cot\left(\frac{\pi}{4}\right) \cdot (-4) \cdot (1 + \cot^2\left(\frac{\pi}{4}\right))$$

$$f'\left(\frac{\pi}{16}\right) = 2 \cdot 1 \cdot (-4) \cdot (1 + 1^2)$$

$$f'\left(\frac{\pi}{16}\right) = -16 //$$

Örnek...11 :

$$f(x) = \sec^2(x^3) \text{ ise } f'(0) = ?$$

$$f(x) = \frac{1}{\cos^2(x^3)} = [\cos(x^3)]^{-2}$$

$$f'(x) = (-2) \cdot [\cos(x^3)]^{-3} \cdot [\cos(x^3)]'$$

$$f'(x) = (-2) \cdot \frac{1}{[\cos(x^3)]^3} \cdot (3x^2) \cdot (-\sin(x^3))$$

$$f'(0) = (-2) \cdot \frac{1}{[\cos(0)]^3} \cdot (3 \cdot 0^2) \cdot (-\sin(0))$$

$$f'(0) = \frac{0}{1^3} = 0 //$$

Örnek...12 :

$$f(x) = \tan(x \cdot \cot^2 x + 1) \text{ ise } f'(x) = ?$$

$$f'(x) = (x \cdot \cot^2 x + 1)' \cdot (1 + \tan^2(x \cdot \cot^2 x + 1))$$

$$f'(x) = [1 \cdot \cot^2 x + x \cdot 2 \cdot \cot x \cdot (-1 - \cot^2 x)] \cdot [1 + \tan^2(x \cdot \cot^2 x + 1)]$$

$$f'(x) = [\cot^2 x + 2x \cdot \cot x \cdot (-1 - \cot^2 x)] \cdot [1 + \tan^2(x \cdot \cot^2 x + 1)] //$$

DEĞERLENDİRME

1) $\frac{d(\sin x \cdot \cos(x^2))}{dx} = ?$

$$\begin{aligned} \frac{d}{dx} (\sin x \cdot \cos(x^2)) &= [\sin x \cdot \cos(x^2)]' \\ &= (\sin x)' \cdot \cos(x^2) + \sin x \cdot [\cos(x^2)]' \\ &= \cos x \cdot \cos(x^2) + \sin x \cdot (2x) \cdot (-\sin(x^2)) \\ &= \cos x \cdot \cos(x^2) - 2x \cdot \sin x \cdot \sin(x^2) // \end{aligned}$$

2) $f(x) = \cos(x^2 - 1)$ ise $f'(0) = ?$

$$\begin{aligned} f'(x) &= 2x \cdot (-\sin(x^2 - 1)) \\ f'(x) &= -2x \cdot \sin(x^2 - 1) \\ f'(0) &= -2 \cdot 0 \cdot \sin(0^2 - 1) = 0 // \end{aligned}$$

3) $f(x) = \sin(\cos 2x)$ ise $f'(\frac{\pi}{8}) = ?$

$$\begin{aligned} f'(x) &= (\cos 2x)' \cdot \cos(\cos 2x) \\ f'(x) &= 2 \cdot (-\sin 2x) \cdot \cos(\cos 2x) \\ f'(x) &= -2 \cdot \sin 2x \cdot \cos(\cos 2x) \\ f'(\frac{\pi}{8}) &= -2 \cdot \sin(2 \cdot \frac{\pi}{8}) \cdot \cos(\cos 2 \cdot \frac{\pi}{8}) \\ &= -2 \cdot \frac{\sqrt{2}}{2} \cdot \cos(\frac{\sqrt{2}}{2}) = -\sqrt{2} \cdot \cos(\frac{\sqrt{2}}{2}) // \end{aligned}$$

4) $f(x) = \tan^2(3x^2 + \frac{\pi}{4})$ ise $f'(0) = ?$

$$\begin{aligned} f'(x) &= 2 \cdot \tan(3x^2 + \frac{\pi}{4}) \cdot [\tan(3x^2 + \frac{\pi}{4})]' \\ f'(x) &= 2 \cdot \tan(3x^2 + \frac{\pi}{4}) \cdot (3x^2 + \frac{\pi}{4})' \cdot (1 + \tan^2(3x^2 + \frac{\pi}{4})) \\ f'(x) &= 2 \cdot \tan(3x^2 + \frac{\pi}{4}) \cdot 6x \cdot (1 + \tan^2(3x^2 + \frac{\pi}{4})) \\ f'(0) &= 2 \cdot \tan(\frac{\pi}{4}) \cdot 6 \cdot 0 \cdot (1 + \tan^2(\frac{\pi}{4})) \\ &= 0 // \end{aligned}$$

5) $f(x) = \sin(\cos(2x))$ ise $\lim_{x \rightarrow \frac{\pi}{16}} \frac{f(x) - f(\frac{\pi}{16})}{x - \frac{\pi}{16}} = ?$

$$\begin{aligned} f'(x) &= (\cos 2x)' \cdot \cos(\cos 2x) \\ f'(x) &= 2 \cdot (-\sin 2x) \cdot \cos(\cos 2x) \\ f'(\frac{\pi}{16}) &= 2 \cdot (-\sin(2 \cdot \frac{\pi}{16})) \cdot \cos(\cos(2 \cdot \frac{\pi}{16})) \\ f'(\frac{\pi}{16}) &= 2 \cdot (-\sin(\frac{\pi}{8})) \cdot \cos(\cos(\frac{\pi}{8})) \\ f'(\frac{\pi}{16}) &= -2 \cdot \frac{\sqrt{2-\sqrt{2}}}{4} \cdot \cos \sqrt{\frac{2+\sqrt{2}}{4}} // \end{aligned}$$

6) $f(x) = \cos 4x$ ise $\lim_{x \rightarrow 0} \left(\frac{f(x) - f(0)}{x} \right) = ? \Rightarrow f'(0)$ sonluyor.

$$\begin{aligned} f'(x) &= 4 \cdot (-\sin 4x) \\ f'(x) &= -4 \cdot \sin 4x \\ f'(0) &= -4 \cdot \sin(0) = 0 // \end{aligned}$$

7) $f(x) = (\sin x - \cos x)^2$ ise $f'(\frac{\pi}{6}) = ?$

$$\begin{aligned} f'(x) &= 2 \cdot (\sin x - \cos x) \cdot (\cos x + \sin x) \\ f'(\frac{\pi}{6}) &= 2 \cdot (\sin \frac{\pi}{6} - \cos \frac{\pi}{6}) \cdot (\cos \frac{\pi}{6} + \sin \frac{\pi}{6}) \\ f'(\frac{\pi}{6}) &= 2 \cdot (\frac{1}{2} - \frac{\sqrt{3}}{2}) \cdot (\frac{\sqrt{3}}{2} + \frac{1}{2}) \\ f'(\frac{\pi}{6}) &= 2 \cdot \frac{1-\sqrt{3}}{2} \cdot \frac{1+\sqrt{3}}{2} = \frac{1^2 - (\sqrt{3})^2}{2} = \frac{-2}{2} // \end{aligned}$$

8) $f(\cot x) = \tan^2(x)$ ise $f'(\frac{\pi}{4}) = ?$

$$\begin{aligned} f(\cot x) &= \left(\frac{1}{\cot x} \right)^2 \\ f'(x) &= \left(\frac{1}{x} \right)^2 \quad \text{NOT} \quad \tan x \cdot \cot x = 1 \\ &\quad \text{orjinali böyle} \quad \tan x = \frac{1}{\cot x} \\ &\quad \text{imisi. } x \text{ yerine} \\ &\quad \text{cot } x \text{ yazılmış.} \end{aligned}$$

$$\begin{aligned} f(x) &= x^{-2} \\ f'(x) &= -2 \cdot x^{-3} \\ f'(\frac{\pi}{4}) &= -2 \cdot \left(\frac{\pi}{4} \right)^{-3} = -2 \cdot \left(\frac{4}{\pi} \right)^3 = \frac{-128}{\pi^3} // \end{aligned}$$

9) $f(x) = \frac{x}{\sin(\frac{x^2}{2})}$ ise $f'(\sqrt{\pi}) = ?$

$$\begin{aligned} f'(x) &= \frac{1 \cdot \sin(\frac{x^2}{2}) - x \cdot (\frac{x^2}{2})' \cdot \cos(\frac{x^2}{2})}{\sin^2(\frac{x^2}{2})} \\ f'(\sqrt{\pi}) &= \frac{\sin(\frac{\pi}{2}) - \sqrt{\pi} \cdot \sqrt{\pi} \cdot \cos(\frac{\pi}{2})}{\sin^2(\frac{\pi}{2})} \\ f'(\sqrt{\pi}) &= \frac{1 - \pi \cdot 0}{1} = 1 // \end{aligned}$$

$$\sin\left(\frac{\pi}{8}\right) = \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{2+\sqrt{2}}{4}}$$

TÜREV - 4

(TRİGONOMETRİK VE TERS TRİGONOMETRİK FONKSİYON TÜREV ALMA KURALLARI)

TERS TRİGONOMETRİK FONKSİYONLARIN TÜREVLERİ

1) $y = \arcsin u \rightarrow y' = \frac{u'}{\sqrt{1-u^2}}$	$y = \arcsin x$ ise $y' = \frac{1}{\sqrt{1-x^2}}$
2) $y = \arccos u \rightarrow y' = \frac{-u'}{\sqrt{1-u^2}}$	$y = \arccos x$ ise $y' = \frac{-1}{\sqrt{1-x^2}}$
3) $y = \arctan u \rightarrow y' = \frac{u'}{1+u^2}$	$y = \arctan x$ ise $y' = \frac{1}{1+x^2}$
4) $y = \text{arccotu} \rightarrow y' = \frac{-u'}{1+u^2}$	$y = \text{arccot} x$ ise $y' = \frac{-1}{1+x^2}$

Örnek...1 :

$$y = \arcsin x \text{ ise } \frac{dy}{dx} = ?$$

$$y' = \frac{1}{\sqrt{1-x^2}} //$$

Örnek...2 :

$$y = \arcsin 3x \text{ ise } \frac{dy}{dx} = ?$$

$$y' = \frac{(3x)'}{\sqrt{1-(3x)^2}} = \frac{3}{\sqrt{1-9x^2}} //$$

Örnek...3 :

$$y = \arctan x \text{ ise } \frac{dy}{dx} = ?$$

$$y' = \frac{1}{1+x^2} //$$

Örnek...4 :

$$y = x^2 \cdot \arctan 2x \text{ ise } \frac{dy}{dx} = ?$$

$$y' = 2x \cdot \arctan 2x + x^2 \cdot \left(\frac{2}{1+(2x)^2} \right) //$$

Örnek...5 :

$$y = \tan(\arccos x) \text{ ise } \frac{dy}{dx} = ?$$

$$y' = (\arccos x)' \cdot (1 + \tan^2(\arccos x))$$

$$y' = \frac{-1}{\sqrt{1-x^2}} \cdot (1 + \tan^2(\arccos x)) //$$

Örnek...6 :

$$y = \cos(\arccos x) \text{ ise } \frac{dy}{dx} = ?$$

$$y' = (\arccos x)' \cdot [-\sin(\arccos x)]$$

$$y' = \frac{-1}{\sqrt{1-x^2}} \cdot (-\sin(\arccos x)) //$$

Örnek...7 :

$$y = \text{arccot}^4(x^3-x) \text{ ise } \frac{dy}{dx} = ?$$

$$y' = 4 \cdot \text{arccot}^3(x^3-x) \cdot [\text{arccot}(x^3-x)]'$$

$$y' = 4 \cdot \text{arccot}^3(x^3-x) \cdot \frac{-(3x^2-1)}{1+(x^3-x)^2}$$

$$y' = \frac{-4 \cdot (3x^2-1) \cdot \text{arccot}^3(x^3-x)}{1+(x^3-x)^2} //$$

TÜREV - 4

(TRIGONOMETRİK VE TERS TRIGONOMETRİK FONKSİYON TÜREV ALMA KURALLARI)

DEĞERLENDİRME

1) $y = \arcsin^2 5x$ ise $\frac{dy}{dx} = ?$

$$y' = 2 \cdot \arcsin 5x \cdot (\arcsin 5x)'$$

$$y' = 2 \cdot \arcsin 5x \cdot \frac{5}{\sqrt{1-(5x)^2}}$$

$$y' = \frac{10 \cdot \arcsin 5x}{\sqrt{1-25x^2}} //$$

2) $y = \arccos^3 2x$ ise $f'(0) = ?$

$$f'(x) = 3 \cdot \arccos^2 2x \cdot (\arccos 2x)'$$

$$f'(x) = 3 \cdot \arccos^2 2x \cdot \frac{-2}{\sqrt{1-4x^2}}$$

$$f'(0) = 3 \cdot \arccos^2(2 \cdot 0) \cdot \frac{-2}{\sqrt{1-4 \cdot 0^2}} = 3 \cdot \arccos^2(0) \cdot (-2)$$

$$= -6 \cdot \left(\frac{\pi}{2}\right)^2 = -\frac{6\pi^2}{4}$$

$$= -\frac{3\pi^2}{2} //$$

3) $y = \arctan^2(2x^3+3x)$ ise $\frac{dy}{dx} = ?$

$$y' = 2 \cdot \arctan(2x^3+3x) \cdot (\arctan(2x^3+3x))'$$

$$y' = 2 \cdot \arctan(2x^3+3x) \cdot \frac{6x^2+3}{1+(2x^3+3x)^2} //$$

HATA! Burada sorulması daha iyi olur. (1 tanımsız yapıyor)

4) $y = \tan(\arccos x)$ ise $f'(1) = ?$

$$f'(x) = \frac{(\arccos x)'}{1 + (\arccos x)^2} = \frac{\frac{-1}{\sqrt{1-x^2}}}{1 + (\arccos x)^2}$$

$$f'(x) = \frac{-1}{\sqrt{1-x^2} \cdot [1 + (\arccos x)^2]}$$

$$f'(0) = \frac{-1}{\sqrt{1-0^2} \cdot [1 + (\arccos(0))^2]}$$

$$f'(0) = \frac{-1}{1 \cdot [1 + (\frac{\pi}{2})^2]} = \frac{-1}{1 + \frac{\pi^2}{4}} = \frac{-1}{\frac{4+\pi^2}{4}}$$

$$f'(0) = \frac{-4}{4+\pi^2} //$$

5) $y = \operatorname{arccot}(\frac{2}{x^2})$ ise $f'(1) = ?$

$$f'(x) = \frac{-\left(\frac{2}{x^2}\right)'}{1 + \left(\frac{2}{x^2}\right)^2} = \frac{-\left(\frac{0 \cdot x^2 - 2 \cdot 2x}{(x^2)^2}\right)}{1 + \left(\frac{2}{x^2}\right)^2} = \frac{-\frac{(-4x)}{x^4}}{1 + \left(\frac{2}{x^2}\right)^2}$$

$$f'(x) = \frac{4}{x^3 \cdot \left(1 + \left(\frac{2}{x^2}\right)^2\right)}$$

$$f'(1) = \frac{4}{1^3 \cdot \left(1 + \left(\frac{2}{1^2}\right)^2\right)} = \frac{4}{(1+4)} = \frac{4}{5} //$$

6) $y = \arcsin(\arccos x)$ ise $f'\left(\frac{\sqrt{3}}{2}\right) = ?$

$$f'(x) = \frac{(\arccos x)'}{\sqrt{1 - (\arccos x)^2}} = \frac{\frac{-1}{\sqrt{1-x^2}}}{\sqrt{1 - (\arccos x)^2}}$$

$$f'\left(\frac{\sqrt{3}}{2}\right) = \frac{\frac{-1}{\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}}}{\sqrt{1 - (\arccos \frac{\sqrt{3}}{2})^2}} = \frac{\frac{-1}{\sqrt{1-\frac{3}{4}}}}{\sqrt{1 - \left(\frac{\pi}{6}\right)^2}}$$

$$f'\left(\frac{\sqrt{3}}{2}\right) = \frac{\frac{-1}{\sqrt{\frac{1}{4}}}}{\sqrt{1 - \frac{\pi^2}{36}}} = \frac{-2}{\sqrt{\frac{36-\pi^2}{36}}} = \frac{-2}{1} \cdot \frac{6}{\sqrt{36-\pi^2}}$$

$$f'\left(\frac{\sqrt{3}}{2}\right) = \frac{-12}{\sqrt{36-\pi^2}} //$$

7) $y = (x^2+3x) \arctan x$ ise $f'(1) = ?$

$$f'(x) = (2x+3) \cdot \arctan x + (x^2+3x) \cdot \frac{1}{1+x^2}$$

$$f'(1) = (2 \cdot 1 + 3) \cdot \arctan(1) + (1^2 + 3 \cdot 1) \cdot \frac{1}{1+1^2}$$

$$f'(1) = 5 \cdot \frac{\pi}{4} + 4 \cdot \frac{1}{2}$$

$$f'(1) = \frac{5\pi}{4} + 2$$

$$f'(1) = \frac{5\pi+8}{4} //$$