

LİMİT - 9

(1° BELİRSİZLİĞİ)

formel $(1+f(x))^{g(x)}$ e uyması için

LİMİT SÜREKLİLİK BÖLÜM 9

1° BİÇİMİNDEKİ BELİRSİZLİKLER

$1^\infty = 1$ dir. 1^∞ belirsizliğiyle ifade edilen aslında $(1,00001\dots)^\infty$ şeklindedir. Kolaylık olsun diye 1^∞ yazılır.

$\lim_{x \rightarrow a} (1+f(x))^{g(x)}$ ifadesinde

$x \rightarrow a$

$\lim_{x \rightarrow a} (f(x))=0$ ve $\lim_{x \rightarrow a} (g(x))=\infty$ olması

$x \rightarrow a$ $x \rightarrow a$

durumunda 1^∞ belirsizliği vardır denir.

Bu belirsizlik türünde

$\lim_{x \rightarrow a} (f(x).g(x))=p$ ise $\lim_{x \rightarrow a} (1+f(x))^{g(x)}=e^p$

olur. (Genel olarak önce limiti alınacak ifadenin logaritmasının limitinden yola çıkılır)

(1°) Örnek...1: $f(x)=\frac{1}{x}$ $g(x)=x$

$$\lim_{x \rightarrow \infty} (1+\frac{1}{x})^x$$

$$\lim_{x \rightarrow \infty} f(x).g(x) = \frac{1}{x} \cdot x$$

$$\lim_{x \rightarrow \infty} 1 = 1$$

$$\lim_{x \rightarrow \infty} (1+\frac{1}{x})^x = e^1 = e$$

(1°) Örnek...2: $f(x)=\frac{4}{x}$ $g(x)=x$

$$\lim_{x \rightarrow \infty} (1+\frac{4}{x})^x$$

$$\lim_{x \rightarrow \infty} f(x).g(x) = 4 \text{ olduğundan}$$

$$\lim_{x \rightarrow \infty} (1+\frac{4}{x})^x = e^4$$

(1°) Örnek...3: $f(x)=\frac{4x}{x^2} = \frac{4}{x}$ $g(x)=2x-7$

$$\lim_{x \rightarrow \infty} (1+\frac{4x}{x^2})^{2x-7}$$

$$\lim_{x \rightarrow \infty} f(x).g(x) = \frac{4}{x} \cdot (2x-7)$$

$$\lim_{x \rightarrow \infty} \frac{8x-28}{x} = \frac{8}{1}$$

$$\lim_{x \rightarrow \infty} (1+\frac{4x}{x^2})^{2x-7} = e^8$$

(1°) Örnek...4:

$$\lim_{x \rightarrow \infty} \left(\frac{x+4}{x+3}\right)^{5x-7} = \lim_{x \rightarrow \infty} \left(\frac{x+3}{x+3} + \frac{1}{x+3}\right)^{5x-7}$$

yazılır.

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x+3}\right)^{5x-7}$$

$$f(x)=\frac{1}{x+3} \quad g(x)=5x-7$$

$$\lim_{x \rightarrow \infty} f(x).g(x) = \frac{5x-7}{x+3}$$

$$\lim_{x \rightarrow \infty} \frac{5x-7}{x+3} = \frac{5}{1}$$

(1°) Örnek...5:

$\lim_{x \rightarrow \infty} (2-\frac{x}{x-2})^{9x-2}$ formele göre $(1+f(x))$ olmalı.

$$\lim_{x \rightarrow \infty} \left(1+1-\frac{x}{x-2}\right)^{9x-2} = \lim_{x \rightarrow \infty} \left(1+\frac{1-\frac{x}{x-2}}{1}\right)^{9x-2}$$

$$\lim_{x \rightarrow \infty} \left(1+\frac{-2}{x-2}\right)^{9x-2}$$

$$f(x)=\frac{-2}{x-2} \quad g(x)=9x-2$$

$$\lim_{x \rightarrow \infty} f(x).g(x)$$

$$\lim_{x \rightarrow \infty} \frac{-2}{x-2} \cdot (9x-2)$$

$$\lim_{x \rightarrow \infty} \frac{-18x+4}{x-2} = \frac{-18}{1}$$

$$\lim_{x \rightarrow \infty} \left(2-\frac{x}{x-2}\right)^{9x-2} = e^{-18}$$

Örnek...6:

$$\lim_{x \rightarrow \infty} x \cdot \ln\left(\frac{x-2}{x+2}\right) = \lim_{x \rightarrow \infty} \ln\left(\frac{x-2}{x+2}\right)^x$$

$$= \lim_{x \rightarrow \infty} \ln\left(\frac{x+2-4}{x+2}\right)^x = \lim_{x \rightarrow \infty} \ln\left(\frac{x+2}{x+2} + \frac{-4}{x+2}\right)^x$$

$$= \ln\left(\lim_{x \rightarrow \infty} \left(1+\frac{-4}{x+2}\right)^x\right)$$

$$= \ln(e^{-4})$$

$$= -4$$

$$f(x)=\frac{-4}{x+2} \quad g(x)=x$$

$$\lim_{x \rightarrow \infty} f(x).g(x)$$

$$\lim_{x \rightarrow \infty} \frac{-4}{x+2} \cdot x = \frac{-4}{1}$$

$$\lim_{x \rightarrow \infty} x \cdot \ln\left(\frac{x-2}{x+2}\right) = -4$$

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DEĞERLENDİRME

$$f(x) = \frac{12}{x} \quad g(x) = \frac{x}{2}$$

$$1) \lim_{x \rightarrow \infty} \left(1 + \frac{12}{x}\right)^{\frac{x}{2}}$$

$$\lim_{x \rightarrow \infty} f(x) \cdot g(x) = \lim_{x \rightarrow \infty} \frac{12}{x} \cdot \frac{x}{2} = 6$$

$$= e^6 //$$

$$2) \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{\frac{2x}{3}} = e^{-\frac{2}{3}} //$$

$$f(x) = -\frac{1}{x} \quad g(x) = \frac{2x}{3}$$

$$\lim_{x \rightarrow \infty} \left(-\frac{1}{x}\right) \cdot \left(\frac{2x}{3}\right)$$

$$\lim_{x \rightarrow \infty} -\frac{2x}{3x} = -\frac{2}{3}$$

$$3) \lim_{x \rightarrow \infty} \left(1 + \frac{3}{4x^2}\right)^{2x^2-7} = e^{\frac{3}{2}} //$$

$$f(x) = \frac{3}{4x^2} \quad g(x) = 2x^2 - 7$$

$$\lim_{x \rightarrow \infty} f(x) \cdot g(x)$$

$$\lim_{x \rightarrow \infty} \frac{3}{4x^2} \cdot (2x^2 - 7)$$

$$\lim_{x \rightarrow \infty} \frac{6x^2 - 21}{4x^2} = \frac{6}{4} = \frac{3}{2}$$

$$4) \lim_{x \rightarrow \infty} \left(3 - \frac{2}{x^2}\right)^{(2x-3)^2} = \left(3 - \frac{2}{\infty}\right)^{\infty}$$

Burada 1° belirsizliği yok. Direk bulunabilir.

$$= 3^{\infty} = \infty //$$

$$5) \lim_{x \rightarrow \infty} \left(\frac{2x^2+3x+4}{2x+3}\right)^{4x-3} = \lim_{x \rightarrow \infty} \left(\frac{x(2x+3)}{2x+3} + \frac{4}{2x+3}\right)^{4x-3}$$

$$= \lim_{x \rightarrow \infty} \left(x + \frac{4}{2x+3}\right)^{4x-3}$$

$$= (\infty + 0)^{\infty}$$

$$= \infty //$$

ifadenin $(1+f(x))^{g(x)}$ ke ben yazılması gerekir.

$$6) \lim_{x \rightarrow \infty} \left(2 - \frac{x^2}{x^2+x-1}\right)^{7x+3}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1 - \frac{x^2}{x^2+x-1}}{1}\right)^{7x+3} = \lim_{x \rightarrow \infty} \left(1 + \frac{x^2+x-1-x^2}{x^2+x-1}\right)^{7x+3}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{x-1}{x^2+x-1}\right)^{7x+3} = e^7 //$$

$$f(x) = \frac{x-1}{x^2+x-1} \quad g(x) = 7x+3$$

$$\lim_{x \rightarrow \infty} f(x) \cdot g(x) = \frac{7x^2 - 6x - 3}{x^2+x-1} = \frac{7}{1}$$

$$7) \lim_{x \rightarrow \infty} (x-2) \cdot \ln\left(\frac{x-2}{x}\right)$$

$$\lim_{x \rightarrow \infty} \ln\left(\frac{x-2}{x}\right)^{x-2} = \lim_{x \rightarrow \infty} \ln\left(\frac{x}{x} + \frac{-2}{x}\right)^{x-2}$$

$$= \ln\left[\lim_{x \rightarrow \infty} \left(1 + \frac{-2}{x}\right)^{x-2}\right] = \ln[e^{-2}]$$

$$= -2 //$$

$$f(x) = -\frac{2}{x} \quad g(x) = x-2$$

$$\lim_{x \rightarrow \infty} f(x) \cdot g(x) = \frac{-2x+4}{x} = -\frac{2}{1}$$