

LİMİT - 8

(0, ∞) BELİRSİZLİĞİ

LİMİT SÜREKLİLİK BÖLÜM 8

0.∞ BİÇİMİNDEKİ BELİRSİZLİKLER

$\lim_{x \rightarrow a} (f(x) \cdot g(x)) = 0 \cdot \infty$ belirsizliği olması durumunda $\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \lim_{x \rightarrow a} \left(\frac{f(x)}{\frac{1}{g(x)}} \right)$ durumuna getirilerek belirsizlik 0/0 veya ∞/∞ biçimine dönüştürülür ve limit aranır.

(0, ∞)

Örnek...1:

$$\lim_{x \rightarrow 0} (x \cdot \cot x) = ? \quad x \cdot \frac{\cos x}{\sin x} = \frac{\cos x}{\frac{\sin x}{x}}$$

$$\lim_{x \rightarrow 0} \frac{\cos x}{\frac{\sin x}{x}} = \frac{\lim_{x \rightarrow 0} \cos x}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{\cos 0^\circ}{1} = \frac{1}{1} = 1 //$$

(0, ∞)

Örnek...2:

$$\lim_{x \rightarrow 0} ((1 - \cos x) \cdot \cot x) = ?$$

$$= \frac{(1 + \cos x) \cdot (1 - \cos x) \cdot \cos x}{(1 + \cos x) \sin x}$$

$$= \frac{1^2 - \cos^2 x \cdot \cos x}{1 + \cos x \sin x}$$

$$= \frac{\sin^2 x \cdot \cos x}{(1 + \cos x) \sin x} = \lim_{x \rightarrow 0} \frac{\sin x \cdot \cos x}{1 + \cos x}$$

(0, ∞)

Örnek...3:

$$\lim_{x \rightarrow \frac{\pi}{2}} \left(\left(x - \frac{\pi}{2} \right) \cdot \tan x \right) = ?$$

$$\left(x - \frac{\pi}{2} \right) \cdot \frac{\sin x}{\cos x} = \sin x \cdot \frac{x - \frac{\pi}{2}}{\cos x}$$

$$= \sin x \cdot \frac{-\left(\frac{\pi}{2} - x \right)}{\sin\left(\frac{\pi}{2} - x \right)} = -\sin x \cdot \frac{\left(\frac{\pi}{2} - x \right)}{\sin\left(\frac{\pi}{2} - x \right)}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} -\sin x \cdot \frac{\frac{\pi}{2} - x}{\sin\left(\frac{\pi}{2} - x \right)} =$$

$$-\sin \frac{\pi}{2} \cdot 1 = -1 \cdot 1 = -1 //$$

(0, ?) belirsizlik

Örnek...4:

$$\lim_{x \rightarrow 2} \left((x-2) \cdot \sin\left(\frac{1}{2-x} \right) \right) = ? \quad \frac{\sin\left(\frac{1}{2-x} \right)}{\frac{1}{x-2}} =$$

$$\lim_{x \rightarrow 2} \left(\frac{\sin\left(\frac{1}{2-x} \right)}{\frac{1}{2-x}} \right) = -1 //$$

(∞ - 0)

Örnek...5:

$$\lim_{n \rightarrow \infty} \left(n \cdot \tan\left(\frac{12}{n} \right) \right) = ? = \frac{12n}{12} \cdot \tan\left(\frac{12}{n} \right)$$

$$\lim_{n \rightarrow \infty} 12 \cdot \frac{\tan\left(\frac{12}{n} \right)}{\frac{12}{n}} = 12 \cdot 1 = 12 //$$

(∞ · 0)

Örnek...6:

$$\lim_{x \rightarrow -\infty} \left(\frac{x}{2} \cdot \sin\left(\frac{2}{x} \right) \right) = ? \quad \lim_{x \rightarrow -\infty} \frac{\sin\left(\frac{2}{x} \right)}{\frac{2}{x}} =$$

$$= 1 //$$

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(0,∞ BELİRSİZLİĞİ)

DEĞERLENDİRME

(∞,0)

$$1) \lim_{x \rightarrow -\infty} (x^2 - x) \cdot \left(\frac{1}{x-1} \right) = ?$$

$$\lim_{x \rightarrow -\infty} x \cdot \cancel{(x-1)} \cdot \frac{1}{\cancel{x-1}} = \lim_{x \rightarrow -\infty} x = -\infty //$$

(0,∞)

$$2) \lim_{x \rightarrow 1} \left((x^3 - 1) \cdot \left(\frac{1}{x^2 - 1} \right) \right) = ?$$

$$\lim_{x \rightarrow 1} \cancel{(x-1)} (x^2 + x + 1) \cdot \frac{1}{\cancel{(x-1)}(x+1)} = (1+1+1) \cdot \frac{1}{2} = \frac{3}{2} //$$

(0,∞)

$$3) \lim_{x \rightarrow 0} (2x \cdot \cot 8x) = ?$$

$$\lim_{x \rightarrow 0} 2x \cdot \frac{1}{\tan 8x} = \lim_{x \rightarrow 0} \frac{2x}{\tan 8x} = \frac{2}{8} = \frac{1}{4} //$$

(0,? belirsiz)

$$4) \lim_{x \rightarrow -\infty} \left(\left(\frac{x}{n} \right) \cdot \sin \left(\frac{m}{x} \right) \right) = ?$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sin \left(\frac{m}{x} \right)}{\left(\frac{x}{n} \right)} &= \frac{\sin \left(\frac{m}{x} \right)}{\frac{1}{n} \cdot \frac{x}{m}} \\ &= \frac{\sin \left(\frac{m}{x} \right)}{\frac{1}{n} \cdot \frac{m}{x}} = \lim_{x \rightarrow -\infty} \frac{m}{n} \cdot \frac{\sin \left(\frac{m}{x} \right)}{\left(\frac{m}{x} \right)} \\ &= \frac{m}{n} \cdot 1 = \frac{m}{n} // \end{aligned}$$

(∞,0)

$$5) \lim_{n \rightarrow \infty} \left((7n+3) \cdot \tan \left(\frac{14}{n} \right) \right) = ?$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(7n \cdot \tan \left(\frac{14}{n} \right) + 3 \cdot \tan \left(\frac{14}{n} \right) \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{14}{n} \cdot 7n \cdot \tan \left(\frac{14}{n} \right) + \lim_{n \rightarrow \infty} 3 \cdot \tan \left(\frac{14}{n} \right) \right) \\ &= \lim_{n \rightarrow \infty} 98 \cdot \frac{\tan \left(\frac{14}{n} \right)}{\frac{14}{n}} + \lim_{n \rightarrow \infty} 3 \cdot \tan \left(\frac{14}{n} \right) \end{aligned}$$

$$6) \lim_{x \rightarrow \infty} \left(\frac{x}{\operatorname{cosec} \frac{3}{x}} \right) = ? \quad \left(\frac{14}{n} \right)$$

$$= 98 \cdot 1 + 3 \cdot 0 = 98 //$$

$$\lim_{x \rightarrow \infty} \frac{x}{\frac{1}{\sin \frac{3}{x}}}$$

$$\lim_{x \rightarrow \infty} \frac{3x}{3} \cdot \sin \frac{3}{x} = \lim_{x \rightarrow \infty} 3 \cdot \frac{\sin \frac{3}{x}}{\frac{3}{x}}$$

$$= 3 \cdot 1 = 3 //$$

$$7) \lim_{x \rightarrow \infty} \left(\frac{x}{4} \cdot \tan \left(\frac{12}{x} \right) \right) = ?$$

$$\lim_{x \rightarrow \infty} \frac{3}{3} \cdot \frac{x}{4} \cdot \tan \left(\frac{12}{x} \right)$$

$$\lim_{x \rightarrow \infty} 3 \cdot \frac{x}{12} \cdot \tan \left(\frac{12}{x} \right)$$

$$\lim_{x \rightarrow \infty} 3 \cdot \frac{\tan \left(\frac{12}{x} \right)}{\frac{12}{x}} = 3 \cdot 1 = 3 //$$

$$8) \lim_{x \rightarrow \infty} \left(\frac{x^2}{4} \cdot \tan^2 \left(\frac{1}{3x} \right) \right) = ? \quad \frac{9}{9} \cdot \frac{x^2}{4} \cdot \tan^2 \left(\frac{1}{3x} \right)$$

$$\lim_{x \rightarrow \infty} \frac{1}{36} \cdot \frac{9x^2}{4} \cdot \tan^2 \left(\frac{1}{3x} \right)$$

$$\lim_{x \rightarrow \infty} \frac{1}{36} \cdot \frac{\tan^2 \left(\frac{1}{3x} \right)}{\left(\frac{1}{3x} \right)^2}$$

$$\lim_{x \rightarrow \infty} \frac{1}{36} \cdot \frac{\tan \frac{1}{3x}}{\frac{1}{3x}} \cdot \frac{\tan \frac{1}{3x}}{\frac{1}{3x}}$$

$$\frac{1}{36} \cdot 1 \cdot 1 = \frac{1}{36} //$$