

TÜREV - 2

KURALLAR-1

TÜREV KURALLARI :

Takip eden türev kurallarının hepsi türevin limit tanımı kullanılarak bulunmuştur. Biz bunlardan sadece ilk ikisinin ispatını vereceğiz.

SABİT FONKSİYONUN TÜREVİ

$f(x)=c \in \mathbb{R}$ ise $f'(x)=0$ olur.

İSPAT

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) = \lim_{h \rightarrow 0} \left(\frac{c - c}{h} \right) = \lim_{h \rightarrow 0} \left(\frac{0}{h} \right) = 0$$

Örnek...1 :

$f(x)=-7$ ise $f'(x)=?$

$$f'(x)=0$$

Örnek...2 :

$f(x)=-3\frac{\pi}{2}$ ise $f'(x)=?$

$$f'(x)=0$$

Örnek...3 :

$f(x)=\sin^2 x + \cos^2 x$ ise $f'(x)=?$

$$f(x)=1 \Rightarrow f'(x)=0$$

KUVVET FONKSİYONUN TÜREVİ

$f(x)=x^n$ ise $f'(x)=n \cdot x^{n-1}$

İSPAT

$$\lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} \right] = \frac{(x+h)^n - x^n}{h}$$

$$\frac{(x+h)^n - x^n}{h} = \frac{(x+h)^{n-1} + (x+h)^{n-2}x + (x+h)^{n-3}x^2 + \dots + x^{n-1}}{1}$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^{n-1} + (x+h)^{n-2}x + (x+h)^{n-3}x^2 + \dots + x^{n-1}}{1} = n \cdot x^{n-1}$$

Örnek...4 :

$f(x)=x$ ise $f'(x)=?$

$$f'(x)=1$$

Örnek...5 :

$f(x)=x^3$ ise $f'(x)=?$

$$f'(x)=3x^2$$

Örnek...6 :

$f(x)=\frac{1}{x}$ ise $f'(x)=?$

$$f(x)=x^{-1}$$
$$f'(x)=-\frac{1}{x^2}$$

Örnek...7 :

$f(x)=\sqrt{x}$ ise $f'(x)=?$

$$f(x)=x^{1/2}$$
$$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$$

Örnek...8 :

$f(x)=\frac{1}{\sqrt[3]{x}}$ ise $f'(x)=?$

$$f(x)=x^{-1/3}$$
$$f'(x)=-\frac{1}{3}x^{-4/3}$$

UYARI

f türevlenebilen bir fonksiyon ve $c \in \mathbb{R}$ ise $(c \cdot f)' = c \cdot f'$ olur.

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Örnek...9 :

$$f(x)=9x^3 \text{ ise } f'(x)=?$$

$$f'(x) = 27x^2$$

Örnek...10 :

$$f(x)=\frac{5}{3}\sqrt[3]{x^2} \text{ ise } f'(x)=? \quad \frac{5}{3} x^{2/3}$$

$$f'(x) = \frac{5}{3} \cdot \frac{2}{3} \cdot x^{-1/3} \\ = \frac{10}{9\sqrt[3]{x}}$$

Örnek...11 :

$$f(x)=\frac{12}{\sqrt[3]{x}} \text{ ise } f'(x)=?$$

$$f(x) = 12 \cdot x^{-1/3} \\ f'(x) = 12 \cdot \left(-\frac{1}{3}\right) x^{-4/3} \\ = \frac{-4}{x^3\sqrt[3]{x}} \quad \text{Oy}$$

TOPLAM VEYA FARKIN TÜREVİ

f ve g türevlenebilen iki fonksiyon ise $f \oplus g$ de f ve g nin türevlenebildiği noktalarda türevlenebilir ve $(f \oplus g)' = f' \oplus g'$

Örnek...12 :

$$f(x)=x^3+x^2 \text{ ise } f'(x)=?$$

$$f'(x) = 3x^2 + 2x$$

Örnek...13 :

$$f(x)=3x^7-2x^6 \text{ ise } f'(x)=?$$

$$f'(x) = 21x^6 - 12x^5$$

Örnek...14 :

$$f(x)=6x^3-7x^2+\frac{1}{x} \text{ ise } f'(1)=?$$

$$f'(x) = 18x^2 - 14x - \frac{1}{x^2} \\ f'(1) = 18 - 14 - 1 = 3 \quad \text{Oy}$$

Örnek...15 :

$$f(x)=ax^b+bx^a \text{ ise } f'(x)=?$$

$$f'(x) = abx^{b-1} + abx^{a-1}$$

Örnek...16 :

$$f(x)=(x^3+x^2)(4x-3) \text{ ise } f'(x)=?$$

$$f(x) = 4x^4 - 3x^3 + 4x^3 - 3x^2 \\ f(x) = 4x^4 + x^3 - 3x^2 \\ f'(x) = 16x^3 + 3x^2 - 6x$$

Örnek...17 :

$$f(x)=\frac{4x^3-3x^2-6x+5}{x^2} \text{ ise } f'(x)=?$$

$$f(x) = 4x - 3 - \frac{6}{x} + \frac{5}{x^2} \\ f'(x) = 4 + \frac{6}{x^2} - \frac{10}{x^3}$$

Örnek...18 :

f(x) polinom fonksiyon ve

$$f(x)+f'(x)=2x^2-3x+2 \text{ ise } f(0)=?$$

$$f(x) = ax^2 + bx + c \\ + f'(x) = 2ax + b \\ \hline 2x^2 + x(b+2a) + b+c = 2x^2 - 3x + 2 \\ 2 = 2 \quad b+4 = -3 \quad -7+c = 2 \\ b = -7 \quad c = 9$$

$f(0) = c = 9$

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Örnek...19 :

$$f(x) = \sum_{k=0}^{30} (-x)^{1+k} \text{ ise } f'(1) = ?$$

$$\begin{aligned} f(x) &= -x + x^2 - x^3 + x^4 \dots - x^{31} \\ f'(x) &= -1 + 2x - 3x^2 + 4x^3 \dots - 31x^{30} \\ f'(1) &= \underbrace{-1 + 2 - 3 + 4 \dots}_{15 \cdot 1} - 31 \\ &= \underline{\underline{-16}} \end{aligned}$$

Örnek...20 :

$$f(x) = \sum_{i=1}^{30} \left(\frac{1}{x}\right)^i \text{ ise } f'(1) = ?$$

$$\begin{aligned} f(x) &= x^{-1} + x^{-2} + \dots + x^{-30} \\ f'(x) &= -x^{-2} - 2x^{-3} + \dots - 30x^{-31} \\ f'(1) &= -1 - 2 - 3 \dots - 30 \\ &= \frac{-30 \cdot 31}{2} = \underline{\underline{-465}} \end{aligned}$$

Örnek...21 :

$$f(x) = \sum_{n=0}^2 \left(\frac{x^n}{n!}\right) \text{ ise } f'(1) = ?$$

$$\begin{aligned} f(x) &= 1 + x + \frac{x^2}{2} \\ f'(x) &= 1 + x \\ f'(1) &= 1 + 1 \\ &= \underline{\underline{2}} \end{aligned}$$

ÇARPIMIN TÜREVİ

f ve g iki fonksiyon olmak üzere , f . g fonksiyonu da f ve g nin türevlenebildiği noktalarda türevlenebilir ve

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Örnek...22 :

$$f(x) = x^2(4x-3) \text{ ise } f'(x) = ?$$

$$\begin{aligned} f'(x) &= 4x^3 - 3x^2 \\ f'(x) &= 12x^2 - 6x \end{aligned}$$

Örnek...23 :

$$f(x) = (x^2 - x) \cdot (x^2 + 1) \text{ ise } f'(-1) = ?$$

$$\begin{aligned} f'(x) &= (2x-1)(x^2+1) + 2x(x^2-x) \\ f'(-1) &= -3 \cdot 2 + (-2) \cdot (1+1) \\ &= -6 - 4 = \underline{\underline{-10}} \end{aligned}$$

Örnek...24 :

$$f(x) = (x^2 + 2x + 4) \cdot (x - 2) \text{ ise } f'(10) = ?$$

$$\begin{aligned} f'(x) &= 3x^2 \\ f'(10) &= 3 \cdot 10^2 = 100 \end{aligned}$$

Örnek...25 :

$$f(x) = (x-1)(x-2)(x-3) \text{ ise } f'(4) = ?$$

$$\begin{aligned} f(x) &= (x^2 - 3x + 2)(x-3) \\ f'(x) &= (2x-3)(x-3) + (x^2-3x+2) \\ f'(4) &= 5 \cdot 1 + 16 - 12 + 2 \\ &= 5 + 6 \\ &= \underline{\underline{11}} \end{aligned}$$

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Örnek 26

$f(x)=(x-1)(x-2)(x-3)$ ise $f'(3)$ kaçtır?

$$f(x)=[(x-1)(x-2)].(x-3)$$

$$f'(x)=g(x).(x-3)+[(x-1)(x-2)]$$

$$f'(3)=0+2.1=2$$

BÖLÜMÜN TÜREVİ

f ve g iki fonksiyon olmak üzere $\frac{f}{g}$ de f ve g nin türevlenebildiği noktalarda türevlenebilir ve

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x).g(x) - f(x).g'(x)}{g^2(x)}$$

Örnek...27 :

$f(x)=\frac{2x^2+1}{1+x}$ ise $f'(x)=?$

$$f'(x) = \frac{4x(1+x) - 1.(2x^2+1)}{(1+x)^2}$$
$$= \frac{2x^2+4x-1}{(x+1)^2}$$

Örnek...28 :

$f(x)=\frac{3x-1}{x+5}$ ve $f^{-1}(x)=h(x)$ ise $h'(x)=?$

$$h(x) = \frac{-5x-1}{x-3}$$

$$h'(x) = \frac{-5(x-3) - 1.(-5x-1)}{(x-3)^2}$$

$$h'(x) = \frac{16}{(x-3)^2}$$

Örnek...29 :

$f(x)=\frac{3x^5-5x^3+1}{x^3-1}$ ise $f'(0)=?$

$$f'(x) = \frac{(15x^4 - 15x^2)(x^3-1) - 3x^2.(3x^5-5x^3+1)}{(x^3-1)^2}$$
$$= \frac{0}{1} = 0$$

Örnek...30 :

$f(x)=\frac{\sqrt{x}}{\sqrt[3]{x+1}}$ ise $f'(1)=?$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2} \cdot (\sqrt[3]{x+1}) - \frac{1}{3}x^{-2/3} \cdot (\sqrt{x})}{(\sqrt[3]{x+1})^2}$$

$$f'(1) = \frac{\frac{1}{2} \cdot 2 - \frac{1}{3} \cdot 1}{(1+1)^2} = \frac{2/3}{4} = \frac{1}{6}$$

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DEĞERLENDİRME

1) $f(x) = \sin^2 2x - \cos^2 2x + \cos 4x$ ise $f'(x) = ?$

$$f(x) = 0$$

$$f'(x) = 0$$

2) $f(x) = \sum_{k=0}^3 (x)^{-k+2}$ ise $f'(1) = ?$

$$f(x) = x^2 + x + 1 + \frac{1}{x}$$

$$f'(x) = 2x + 1 - \frac{1}{x^2}$$

$$f'(1) = 2 + 1 - 1 = 2$$

3) $f(x) = \sqrt{x} \cdot (\sqrt{x} - \sqrt[3]{x})$ ise $f'(64) = ?$

$$f(x) = x - x^{5/6}$$

$$f'(x) = 1 - \frac{5}{6} x^{-1/6}$$

$$f'(64) = 1 - \frac{5}{6} \cdot 2^{-1} = 1 - \frac{5}{12}$$

$$= \frac{7}{12}$$

4) $f(x)$ polinom fonksiyon ve $f(x) \cdot f'(x) = 2x^3 - x^2 + 10x + d$ ise d kaçtır?

$$f(x) = ax^2 + bx + c$$

$$f'(x) = 2ax + b$$

$$(2x^2 + bx + c)(2ax + b)$$

$$2ax^3 + x^2(2b + 2ab) + x(b^2 + 2ac) + bc$$

$$2a = 1 \quad 3b = -1 \quad b = -\frac{1}{3} \quad c = \frac{89}{18}$$

5) $f(x) = (x^3 + x^2)(4x - 3)$ ise $f'(1) = ?$

$$f'(x) = (3x^2 + 2x)(4x - 3) + 4 \cdot (x^3 + x^2)$$

$$f'(1) = 5 \cdot 1 + 4 \cdot 2 = 13$$

6) $f(x) = (x-1)(x-2)(x-3)\dots(x-10)$ ise $f'(4) = ?$

$$f(x) = (x-4) \cdot [(x-1) \dots (x-10)]$$

$$f'(x) = 1 \cdot [(x-1)(x-2)\dots(x-10)] + [(x-1)\dots(x-10)]' \cdot (x-4)$$

$$f'(4) = 3 \cdot 2 \cdot 1 \cdot (-1) \cdot (-2) \cdot \dots \cdot (-6) + 0$$

$$f'(4) = 3! \cdot 6!$$

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7) $f(x) = \frac{3x^2 - x}{x^2 + 1}$ ise $f'(1) = ?$

$$f'(x) = \frac{(6x-1)2x - 2x(3x^2-x)}{(x^2+1)^2}$$

$$f'(1) = \frac{10-4}{4} = \frac{3}{2}$$

8) $x > 0$, $f(x) = \frac{\sum_{k=1}^5 x^{2k}}{\sum_{k=1}^5 x^{2k-1}}$ olduğuna göre $f'(1) = ?$

$$f(x) = \frac{x^2 + x^4 + \dots + x^{10}}{x + x^3 + \dots + x^9}$$

$$f(x) = x$$

$$f'(x) = 1$$

$$f'(1) = 1$$

9) Fonksiyonların önce terslerini sonra türevlerini bulunuz

a) $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x+5$ ise $(f^{-1})'(x) = ?$

$$f^{-1}(x) = \frac{x-5}{3} \quad (f^{-1})'(x) = \frac{1}{3}$$

b) $f: \mathbb{R} - \{2\} \rightarrow \mathbb{R} - \{3\}$, $f(x) = \frac{3x-5}{x-2}$ ise

$$(f^{-1})'(1) = ?$$

$$f^{-1}(x) = \frac{2x-5}{x-3}$$

$$(f^{-1})'(x) = \frac{2(x-3) - 1 \cdot (2x-5)}{(x-3)^2}$$
$$= \frac{-1}{(x-3)^2}$$

c) $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \sqrt[3]{x^3+2}$ ise $(f^{-1})'(x) = ?$

$$y = \sqrt[3]{x^3+2}$$

$$y^3 - 2 = x^3$$

$$x = \sqrt[3]{y^3-2}$$

$$f^{-1}(x) = \sqrt[3]{x^3-2}$$

$$(f^{-1})'(x) = \frac{1}{3} (x^3-2)^{-2/3} \cdot (3x^2)$$

Not $y = u^n \Rightarrow y' = n \cdot u^{n-1} \cdot u'$
(bir sonraki bölümde anlatılacak!)