

TRİGONOMETRİ - 1

YAY TOPLAM VE FARKI

YAY TOPLAM VE FARKININ TRİGONOMETRİK ORANLARI

$$1) \sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$$

$$2) \sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$$

$$3) \cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

$$4) \cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$$

Örnek...1 :

İfadelerin değerini hesaplayınız?

$$a) \cos(15^\circ) = \cos(60-45) \\ \cos 60 \cos 45 + \sin 60 \sin 45 \\ \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$b) \sin(15^\circ) = \sin(60-45) = \sin 60 \cos 45 - \sin 45 \cos 60 \\ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Örnek...2 :

Bir ABC üçgeninde $\cos A \cdot \cos B - \sin A \cdot \sin B = \cos(A+B)$ işleminin C türünden eşiti nedir?

$$\cos(A+B) = \cos(180-C) = -\cos C$$

Örnek...3 :

$$\frac{\sin 27 \cdot \cos 33 + \sin 33 \cdot \cos 27}{\sin 16 \cdot \sin 14 - \cos 14 \cdot \cos 16} \text{ işleminin sonucu kaçtır?}$$

$$\frac{\sin 60}{-\cos 30} = -1$$

$$\frac{\sin 22 \cdot \cos 8 + \cos 82 \cdot \cos 22}{\cos 34 \cos 11 - \cos 56 \cos 79} \text{ işleminin sonucu kaçtır?}$$
$$\frac{\sin(22+8)}{\cos(34+11)} = \frac{\sin 30}{\cos 45} = \frac{1/2}{1/\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$5) \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$$

$$6) \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$$

$$7) \cot(a+b) = \frac{1}{\tan(a+b)}$$

Örnek...5 :

İfadelerin değerini hesaplayınız?

$$a) \tan(15^\circ) = \tan(60-45) = \frac{\tan 60 - \tan 45}{1 + \tan 60 \cdot \tan 45} \\ = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} = 2 - \sqrt{3}$$

$$b) \tan(75^\circ) = \tan(45+30) = \frac{\tan 45 + \tan 30}{1 - \tan 45 \cdot \tan 30} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} \\ = 2 + \sqrt{3}$$

$$c) \cot\left(\frac{11\pi}{12}\right) = \cot(165^\circ) = -\cot 15^\circ = -\tan 75^\circ = -(2 + \sqrt{3})$$

Örnek...6 :

$\tan x = u$ olmak üzere $\cot(135^\circ - x)$ u türünden nedir?

$$\cot(90+45-x) = -\tan(45-x) \\ = -\frac{\tan 45 - \tan x}{1 + \tan 45 \tan x} = -\frac{1-u}{1+u} \\ = \frac{u-1}{u+1} \checkmark$$

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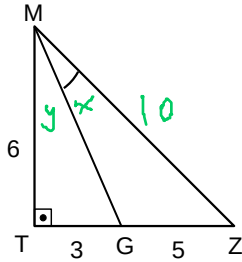
Örnek...7 :

MTZ bir üçgen,
|TG|=3br, |GZ|=5br
|MT|=6br olduğuna göre

$\tan(\widehat{GMZ})$ kaçtır?

$$\tan(x+y) = \frac{\frac{8}{6}}{\frac{4}{3}} = \frac{4}{3}$$

$$\tan y = \frac{3}{6} = \frac{1}{2}$$



$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\tan x + \frac{1}{2}}{1 - \tan x \cdot \frac{1}{2}} = \frac{4}{3}$$

$$\tan x = \frac{1}{2}$$

Örnek...8 :

ABCD dikdörtgen

|AB|=|AE|

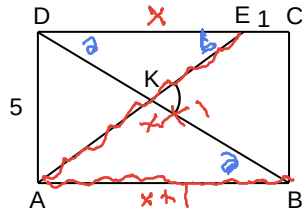
|AD|=5 br

|CE|=1 br

olduğuna göre,

$\tan(\widehat{BKE})$ kaçtır?

$$x=12$$



$$\tan(\widehat{BKE}) = \tan(2+x) = \frac{\tan 2 + \tan x}{1 - \tan 2 \tan x}$$

$$= \frac{\frac{5}{13} + \frac{5}{12}}{1 - \frac{5}{13} \cdot \frac{5}{12}}$$

$$= \frac{125}{131}$$

Örnek...9 :

$$\frac{\sqrt{3} \cdot \cos 20 - \sin 20}{\cos 17 \cdot \cos 33 + \cos 107 \cdot \sin 33}$$

$$\frac{\frac{\sin 60}{\cos 60} \cdot \cos 20 - \sin 20}{\cos 60 \cdot (\cos 33 + \sin 33)} = \frac{\sin(60-20)}{\cos 60 \cdot (\cos 33 + \sin 33)}$$

$$= \frac{\sin 40}{\frac{1}{2} \cdot \cos 50}$$

$$= 2/\cos 10$$

YARIM AÇI BAĞINTILARI

$$1) \sin(2x) = 2\sin x \cdot \cos x$$

$$2) \cos(2x) = 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$= \cos^2 x - \sin^2 x$$

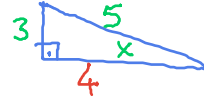
$$3) \tan(2x) = \frac{2 \cdot \tan x}{1 - \tan^2 x}$$

$$4) \cot(2x) = \frac{1}{\tan 2x}$$

Örnek...1 :

$$\sin x = \frac{3}{5} \text{ ise } \tan 2x = ?$$

$$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \cdot \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2} = \frac{24}{7}$$



Örnek...2 :

$$\left(\cos x - \sin x = \frac{2}{3}\right)^2 \text{ ise } \sin 2x \text{ kaç olabilir?}$$

$$\cos^2 x - 2\sin x \cos x + \sin^2 x = \frac{4}{9}$$

$$1 - \sin 2x = \frac{4}{9}$$

$$\sin 2x = \frac{5}{9}$$

Örnek...3 :

$$\frac{2 \sin 15^\circ \cdot \cos 15^\circ}{2} = ?$$

$$\frac{\sin 30}{2} = \frac{1/2}{2} = \frac{1}{4}$$

Örnek...4 :

$$\sin^2 15^\circ - \cos^2 15^\circ = ?$$

$$-(\cos^2 15 - \sin^2 15)$$

$$-\cos 30 = -\frac{\sqrt{3}}{2}$$

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Örnek...5 :

$\frac{\cos 2x}{\cos x - \sin x} = \frac{1}{2}$ ise $\tan 2x$ kaç olabilir?

$$\frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \frac{1}{2}$$

$$\left[\cos x + \sin x = \frac{1}{2} \right]^2$$

$$\cos^2 x + 2 \sin x \cos x + \sin^2 x = \frac{1}{4}$$

$$1 + \sin 2x = \frac{1}{4}$$

$$\sin 2x = -\frac{3}{4}$$

$$\tan 2x = \frac{\pm 3}{\sqrt{7}}$$



Örnek...6 :

$\frac{\sin 99^\circ}{\sin 27^\circ} + \frac{\cos 99^\circ}{\cos 27^\circ}$ işleminin sonucu kaçtır?

$$\frac{\sin 99 \cos 27 + \cos 99 \sin 27}{\sin 27 \cos 27 \cdot 2/2}$$

$$\frac{\sin 126}{\sin 54} = \frac{\sin 54}{\sin 54} = 2$$

Örnek...7 :

$\sin 18^\circ = p$ ise $\cos^4 18^\circ - \sin^4 18^\circ$ ifadesinin p cinsinden eşitini bulunuz?

$$\cos^4 18 - \sin^4 18 = (\cos^2 - \sin^2)(\cos^2 + \sin^2)$$

$$\downarrow$$

$$\cos 36 = 1 - 2 \sin^2 18$$

$$= 1 - 2p^2$$

Örnek...8 :

$\tan 70^\circ = y$ ise $\cos 40^\circ$ ifadesinin x ve y cinsinden eşitini bulunuz?

$$\cot 20 = y$$

$$\cos 40 = 2 \cos^2 20 - 1$$

$$= 2 \cdot \left(\frac{y}{\sqrt{1+y^2}} \right)^2 - 1$$

$$= \frac{2y^2 - (1+y^2)}{1+y^2}$$

$$= \frac{y^2 - 1}{y^2 + 1}$$



Örnek...9 :

$\tan\left(\frac{x}{2}\right) = t$ ise $\cos x$ ifadesinin t cinsinden eşitini bulunuz?

$$\cos x = 2 \cos^2 \frac{x}{2} - 1$$

$$= 2 \cdot \frac{1}{1+t^2} - 1$$

$$= \frac{2 - 1 - t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$$

Örnek...10 :

$\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ$ çarpımının eşitini bulunuz?

$$\frac{2 \sin 20 \cos 20 \cos 40 \cos 80}{2 \sin 20} = \frac{2 \sin 40 \cos 40 \cos 80}{2 \cdot 2 \sin 20} = \frac{2 \sin 80 \cos 80}{2 \cdot 4 \sin 20} = \frac{\sin 160}{8 \sin 20} = \frac{1}{8}$$

Örnek...11 :

$\frac{1+\cos 2x}{1-\cos 2x} \cdot \tan^2 x$ işleminin sonucu kaçtır?

$$\frac{1+2\cos^2 x-1}{1-(1-2\sin^2 x)} \cdot \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{2\cos^2 x}{2\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} = 1$$

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DEĞERLENDİRME

- 1) Bir ABC üçgeninde $\cot A = \frac{1}{3}$ ve $\cot B = \frac{1}{4}$ olmak üzere, $\cot(\hat{C})$ kaçtır?

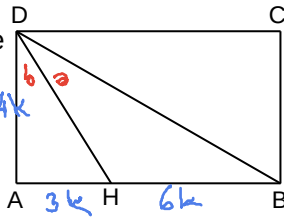
$$\tan A = 3 \quad \tan B = 4$$

$$\tan(C) = \tan(180 - (A+B)) = -\tan(A+B)$$

$$= -\frac{(3+4)}{1-3 \cdot 4} = \frac{-7}{-11}$$

$$\cot C = \frac{11}{7}$$

- 2) ABCD dikdörtgeninde $3|AD|=4|AH|=2|HB|$ ise $\cot(\widehat{HDB})$ kaçtır?



$$\tan(\alpha + \beta) = 9/4$$

$$\tan \alpha = 3/4$$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{3}{4}$$

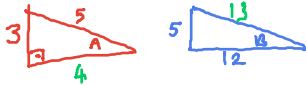
$$\tan \alpha = \frac{3}{4}$$

$$\cot \alpha = \frac{4}{3}$$

$$\frac{\tan \alpha + 3/4}{1 - \tan \alpha \cdot 3/4} = \frac{9}{4}$$

- 3) $\cos\left(\arcsin \frac{3}{5} + \arctan \frac{5}{12}\right) = ?$ $\sin A = 3/5$ $\tan B = 5/12$

$$\cos A \cos B - \sin A \sin B = \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13} = \frac{33}{65}$$



- 4) $\sin 18^\circ = x$, ise $\sin 126^\circ$ ifadesinin x cinsinden eşitini bulunuz?

$$\sin 126 = \sin(90 + 36) = \cos 36 = ?$$

$$\cos 36 = 1 - 2\sin^2 18$$

$$= 1 - 2x^2$$

$$y^a \downarrow a \quad \cos 72 = x \quad 2\cos^2 36 - 1 = x \Rightarrow \cos 36 = \frac{x+1}{2}$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

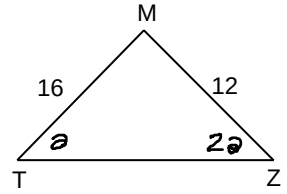
- 5) $\operatorname{cosec} 18^\circ - \sec 36^\circ = ?$

$$\frac{1}{\sin 18} - \frac{1}{\cos 36} = \frac{\cos 36 - \sin 18}{\sin 18 \cos 36}$$

$$= \frac{-2 \sin 54 \cdot \sin(-18)}{\sin 18 \cdot \cos 36}$$

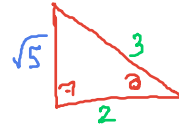
$$= 2$$

- 6) MTZ bir üçgen $m(\widehat{TZM}) = 2 \cdot m(\widehat{ZTM})$ ise $\tan(\widehat{MTZ})$ kaçtır?



(Sinüs teoremi kullanınız)

$$\frac{12}{\sin \alpha} = \frac{16}{\sin 2\alpha} \Rightarrow \cos \alpha = \frac{16}{24} = \frac{2}{3}$$



$$\tan \alpha = \frac{\sqrt{5}}{2}$$

- 7) $\frac{\sqrt{1+\sin 2x}}{\cos^2 x - \sin^2 x} = \frac{3}{2}$ ise $\sin 2x$ kaç olabilir?

$$\frac{\sqrt{1+2\sin x \cos x}}{\cos^2 x - \sin^2 x} = \frac{\sqrt{(c+s)^2}}{c^2 - s^2} = \frac{(c+s)}{(c-s)(c+s)} = \frac{1}{c-s} = \frac{3}{2}$$

$$(c-s = \frac{2}{3})^2$$

$$c^2 - 2cs + s^2 = \frac{4}{9}$$

$$1 - \sin 2x = \frac{4}{9} \rightarrow \sin 2x = \frac{5}{9}$$

- 8) $\arctan\left(\frac{\sin 10 + \sin 50}{\cos 10 + \cos 50}\right) = ?$

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\arctan\left(\frac{2 \sin 30 \cos 20}{2 \cos 30 \cos 20}\right) = \arctan(\tan 30)$$

$$= \frac{\pi}{6}$$